Scientific Professional Quarterly

Hydrualic Structures

http://Jhs.scu.ac.ir

Spring 2013

Volume 1, Issue 1, Pages 54-64

Shahid Chamran University

Chaotic Analysis and Prediction of River Flows

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Abstract

Analyses and investigations on river flow behavior are major issues in design, operation and studies related to water engineering. Thus, recently the application of chaos theory and new techniques, such as chaos theory, has been considered in hydrology and water resources due to relevant innovations and ability. This paper compares the performance of chaos theory with Anfis model and discusses on application case in the context of different interpretations of chaotic behaviour in river flow time series. This study determines the daily flow properties of river Aharchai in during 19 years using the concepts of chaos theory and predicted flows. Reconstruction of state space time series using chaos theory, based on appropriate selection of delay time and embedding dimension. Average mutual Correlation dimension technique has been used for definition of fractal dimension and evaluation of chaos in time series. Results of Evaluations show the fractals dimension of 4 (chaotic low), with a time delay of 65 days and embedding dimension of 13 that can be used for the reconstruction of dynamic state space of river flow. Local prediction algorithm is used for prediction of the time series. The results represent acceptable precision and adequate theory of chaos in flow forecasting of Aharchai River.

Keywords: *embedding dimension, fractal dimension, Aharchai River, Chaos theory, false nearest neighbors*

1 Introduction

Study of river flow is one of the most important cases in designing of water storage structure and management of extreme events such as floods and droughts. The rate of river flow depends on various parameters and the nonlinear relationship between them has caused river behaviour to be dynamic, nonlinear and complex. Previous studies on river flow have led to the development of: (i) traditional stochastic models widely practiced and applied to data with considerable fluctuations; (ii) *distributed hydraulic models* give an insight into catchment processes by their formidable prediction capabilities but require detailed data; (iii) *timeseries analysis*, including chaos theory, catastrophe theory and other artificial intelligence techniques, are successful predictive capabilities, which do not require any data other than time series but are able to provide an into insight the data. After two decades or so of applying time series, the thriving research is yet to be transformed into working modeling practices.

Even though applications of (linear) stochastic approaches are very common in study of complex natural and physical systems, such as hydrological processes, but it is appropriate to use - nonlinear approaches as stochastic approaches might have limitations. The observation of chaotic behavior in hydrological processes supports concerns [1] [2][3]. This paper is focused on chaos theory, compares its performance with the ANFIS model and discusses on the application case in the context of different interpretations of chaotic behavior in river flow time series. Chaos theory is the study of complex systems that, at first glance, appear does not follow the regular laws of science. Chaos theory is one of the most fascinating and promising developments in the late 20th century mathematics and science. It provides a way of making sense out of phenomena such as river flow that seem to be totally without organization or order. A chaotic system is defined as a deterministic system in which small changes in the initial conditions may lead to completely different behavior in the future. The instability, non-periodic behavior, certain systems, being nonlinear, in alongside each other is defined chaotic systems.

Instability, non-periodic behaviour, certain systems, being nonlinear, together is defined the chaotic systems. For the first time Chaos Theory was used by Edward

Lorenz in 1965 in meteorology. Later it has been implemented in all fields of science and empirical issues e.g. mathematics, behavioral, Astronomy, mechanics, physics, mathematics, biology, economics and etc

To date, a lot of attention has been devoted on analysing hydrological processes and elements by means of deterministic chaos approach. For example Domenico and Ghorbani [4], Nagesh Kumar and Dhanya [5], Ghorbani and et al. [6], Islam and Sivakumar [2], Sivakumar and et al. [7], Lisi and Villi [8], Liu et al. [9]. Stehlik [10] and Regonda and et al. [11] have used nonlinear deterministic approaches to detect the presence of chaos and achieve more accurate river flow predictions.

These investigations suggest that characterization (chaotic or stochastic) of river flow should be a necessary first step in any study, as it could provide important information on appropriate approaches for transforming data.

Numerous studies have estimated the level of sea using chaos theory such as Ghorbani and et al. [12], Siek M, Solomatine [13], Khokhlov and et al. [14], Skebede and Travi [15], Bayram and et al. [16], Solomatine and et al. [17].

There are other applications of chaos theory in the various topics that are not discussed in here. In this paper the behavior of river flow (daily river discharge) is forecasted by means of chaos theory.

2 Descriptions of Selected Chaos Models

Chaotic behaviors reflect their internal processes in the time history of one of their single variables, normally referred to as time series, which may therefore bear external signals. A range of nonlinear dynamic methods have specifically been developed to identify chaotic behaviours mainly from time series and this study employs a number of them described below, including stochastic techniques.

2.1) Phase Space Reconstruction by the Average Mutual Information (AMI) Method

One way of characterizing dynamical systems is by the concept of phase space, according to which given a set of physical variables and an analytical model describing their interactions, the dynamics of the system can be represented geometrically by a single point moving along a trajectory, where each of its points corresponds to a state of the system. The delay-embedding method reconstructs phase space from a univariate or multivariate time series which is assumed to be generated by a deterministic dynamical system [18]. The Takens theorem states that the underlying dynamics can be fully recovered by building an m-dimensional space wherein the components of each state vector Y_t, correlated to observed values, which are discrete scalar time series, $X_t=\{x_1, x_2, ...x_N\}$ with N-observed values, with delay coordinates in the m-dimensional phase space:

$$Y_t = \left\{X_t, X_{t-\tau}, X_{t-2\tau}, \ldots, X_{t-(m-1)\tau}\right\} \quad (1)$$

Where τ is referred to as the delay time and, for a digitized time series, it is a multiple of the sampling interval used, and m is termed the embedding dimension. Those systems whose dynamics can be reduced to a set of inherently deterministic behaviours, their trajectories converge towards the subset of the phase space, called the attractor. The reconstruction of phase-space by plotting X_t against $X_{t-(m-1)\tau}$ can show the presence of an attractor as a visual evidence for deterministic chaos in a given time series.

Mathematical approaches for the reconstruction of the phase space diagram of chaotic behaviours may be carried out by one of the following methods: (i) AutoCorrelation Function, ACF (e.g. Holzfuss and Mayer-Kress [19]), (ii) Average Mutual Information, AMI (Fraser and Swinney [20]) or Correlation Integral, CI (e.g. Leibert and Schuster [21]). This study uses the ACF, defined as:

$$\rho_k = \frac{E[(x_t-\mu)(x_{t+k}-\mu)]}{\sqrt{E[(x_t-\mu)^2]E[(x_{t+k}-\mu)^2]}} \quad (2)$$

Where ρ_k is autocorrelation function; E is a functional expression of expectation, x is the observed data. For each value of m there is a value of, ρ_k . Holzfuss and Mayer-Kress [19] recommend the ascertainment of the value of delay time by displaying ρ_k against m, and obtaining its value at its first zero crossing of the autocorrelation function. This method is selected in this study but Schuster [22] recommends the value when ACF is 0.5 or Tsonis [23] when its value is 0.1. Behavior of the autocorrelation function ρ_k as a function of m is indicative of the dynamics of the process controlling the timeseries.

After determining the values of m the phase-space diagram may be reconstructed. The attractor is the geometric description of a single moving point by displaying X_t against $X_{t-(m-1)\tau}$, for which the following outcomes are possible: (i) for a rather periodically regular behavior, the attractor will be a well-defined closed shape; (ii) for stochastic processes, the attractor would look like a cloud of points; and (ii) for a deterministic chaotic behavior, the attractor revolve around a recognizable closed curve but every now and then it would tend to get out of track.

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2.2) Correlation Dimension Method

Correlation dimension is a nonlinear measure of the correlation between pairs lying on the attractor. For time series whose underlying dynamics is chaotic, the correlation dimension gets a finite fractional value, whereas for stochastic systems it is infinite. For an m-dimensional phase space, the correlation function Cm(r) is defined as the fraction of states closer than r, (Grassberger and Procaccia [24]): The correlation dimension method is one of the most widely used methods to determine the presence of chaos, and more specifically to distinguish between low-dimensional and high-dimensional systems. For chaotic systems, the dimension is non-integer and low. The method uses the correlation function to determine the dimension of the attractor in the phase space. For an m-dimensional phase space the correlation function C(r) is given by: (Grassberger and Procaccia [24])

$$C(r) = \lim_{N\to\infty} \frac{2}{N(N-1)} \sum_{i,j=1}^{N} H(r - |Y_i - Y_j|) \qquad (3)$$

where H is the Heaviside step function, for $u = r - |Y_i - Y_j|$ and $u \geq 0$, $H(u) = 1$ and $u \leq 0$, $H(u) = 0$, N is the number of points on the reconstructed attractor, r is the radius of the sphere centered on Y_i or Y_j. An attractor is represented by radius, r, and a non-integer fractal dimension, as follows:

$$C(r) \propto \alpha r^{D_2} \qquad (4)$$

Where α is a constant; and D2 is the correlation exponent or the slope of $\ln C(r)$ versus $\ln(r)$ given by:

$$D_2 = \lim_{r\to 0} \frac{\ln C(r)}{\ln r} \qquad (5)$$

The behavior of D_2 provides one technique for determining the presence of chaos in a time series, such that (i) for stochastic processes, D_2, varies linearly with increasing m, without reaching a saturation value; (ii) for deterministic processes the value of D_2 saturates after a certain value of m.

2.3) Local prediction

A correct phase-space reconstruction in a dimension m facilitates an interpretation of the underlying dynamics in the form of an m-dimensional map, f_T according to

$$Y_{j+T} = f_T(Y_j) \qquad (6)$$

Where Y_j and Y_{J+T} are vectors of dimension m, describing the state of the system at times j (i.e. the current state) and *j+T* (i.e. the future state), respectively. The problem then is to find an appropriate expression for f_T (i.e. F_T). Local approximation entails the subdivision of the f_T domain into many subsets (neighborhoods), each of which identifies some approximations FT, valid only in that same subset. In other words, the dynamics of the system is described step-by-step locally in the phase-space. In this m-dimensional space, prediction is performed by estimating the change of X_i with time. Considering the relation between the points X_t and X_{t+p}, the behavior at a future time p on the attractor is approximated by function F as:

$$X_{t+p} \cong F(X_t) \qquad (7)$$

In this prediction method, the change of X_t with time on the attractor is assumed to be the same as those of nearby points, $(X_{T_h}, h = 1,2,..., n)$. Herein, X_{t+p} is determined by the *d*th order polynomial $F(X_t)$ as follows [25]

$$x_{t+p} \cong f_0 + \sum_{k_1=0}^{m-1} f_{1k_1} X_{t-k_1\tau} + \sum_{\substack{k_2=k_1 \\ k_1=0}}^{m-1} f_{2k_1k_2} X_{t-k\tau_1} X_{t-k_2\tau} + ... + \sum_{\substack{k_d=k_{d-1} \\ . \\ . \\ . \\ k_2=k_1 \\ k_1=0}}^{m-1} f_{dk_1k_2...k_d} X_{t-k_1\tau} X_{t-k_2\tau} ... X_{t-k_d\tau}$$

(8)

Using n of X_{T_h} and $X_{T_{h+p}}$ for which the values are already known, the coefficients, f, are determined by solution of the following equation:

$$X \cong Af \qquad (9)$$

where,

$$X = (X_{T_{1+p}}, X_{T_{2+p}}, ..., X_{T_{n+p}}) \qquad (10)$$

and,

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$$f = (f_0, f_{10}, f_{11}, ..., f_{1(m-1)}, f_{200}, ..., f_{d(m-1)(m-1)...(m-1)}) \quad (11)$$

A is the $n \times (m+d)!/m!d!$ Jacobian matrix which in its explicit form is:

$$A = \begin{bmatrix} 1 & X_{T_1} & X_{T_{1-\tau}} & \dots & X_{T_{1-(m-1)\tau}} & X_{T_1}^2 & \dots & X_{T_{1-(m-1)\tau}}^d \\ 1 & X_{T_2} & X_{T_{2-\tau}} & \dots & X_{T_{2-(m-1)\tau}} & X_{T_2}^2 & \dots & X_{T_{2-(m-1)\tau}}^d \\ \vdots & & & \vdots & & \vdots & & \vdots \\ 1 & X_{T_n} & X_{T_{n-\tau}} & \dots & X_{T_{n-(m-1)\tau}} & X_{T_n}^2 & \dots & X_{T_{n-(m-1)\tau}}^d \end{bmatrix}$$
(12)

In order to obtain a stable solution, the number of rows in the Jacobian matrix A must satisfy:

$$n \geq \frac{(m+d)!}{m!d!} \quad (13)$$

As stated by Porporato and Ridolfi [26], even though in the case F are first degree polynomials, the prediction is nonlinear, because during the prediction procedure every point x(t) belongs to a different neighbourhood and is therefore defined by different expressions for f (Kocak [27]).

3 Adaptive neuro fuzzy inference systems

The Adaptive neuro fuzzy inference system (ANFIS), first introduced by Jang (1993) is capable of approximating any complex processes (Jang et al., [28]).

The integration of the techniques of fuzzy systems and ANN suggests the novel idea of transforming the burden of designing fuzzy systems to the training and learning of the ANN. The ANN provide learning ability to the fuzzy systems, whereas the fuzzy systems offer ANN a structure framework with high level IF-THEN rule thinking and reasoning.

The neuro fuzzy, one form of integration of fuzzy systems and ANN, is a fuzzy system that uses a learning algorithm derived from or inspired by Ann theory to determine its parameters (fuzzy memberships and fuzzy rules) by processing data. In order works, ANFIS aim at providing fuzzy systems with automatic training method of ANN, but without altering their functionality. In an ANFIS, the ANN helps the fuzzy system to elicit membership functions, map fuzzy sets to fuzzy rules, and implement defuzzification.

3.1) Fundamental of neuro fuzzy modeling Originality

3.1.1) Fuzzy modeling

In the last years, fuzzy logic based procedures have proven to be very efficient for analyzing data and modeling the according processes (Zadeh, [29]). Since then the fuzzy logic concept has found a very wide range of application in various domains like: estimation, prediction, control, approximate reasoning, optimization and industrial engineering, etc. Especially they are used, when conventional procedures are getting rather complex and expensive or vague and imprecise information flows directly into the modeling process. With fuzzy logic it is possible to describe available knowledge directly in linguistic terms and according rules.

A general fuzzy inference system (FIS) has four components; fuzzification, fuzzy rule base, fuzzy output engine, and defuzzification. (Tayfur et al. [30]). (Ross [31]).

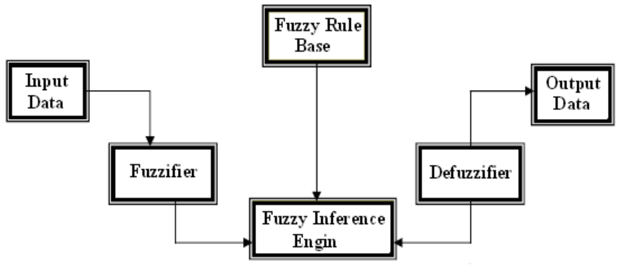

Fig.1 The general structure of the fuzzy inference system

3.1.2) Artificial neural network Models

An ANN, can be defined as a system or mathematical model consisting of many nonlinear artificial neurons running in parallel which can be generated as one or multiple layered. In this study Feed Forward Neural Networks (FFNN) are used for modeling of daily runoff. A FFNN consists of at least three layers, input, output and hidden layers. The number of hidden layers and neurons are determined by trial and error method. The schematic diagram of a FFNN is shown in Fig. 2.

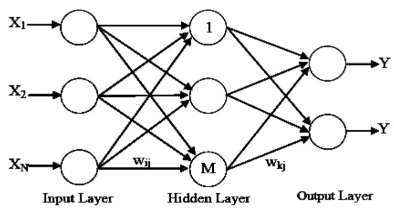

Fig.2 The general structure of a FFNN

3.1.3) Adaptive neuro-fuzzy inference system (ANIS)

The adaptive neuro-fuzzy inference system (ANFIS) is a universal approximator and, as such, is enable of approximating any real continuous function on a compeal set to any degree of accuracy [28].

The ANFIS is functionally equivalent to FIS [28]. Below, the hybrid learning algorithm, which combines

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gradient descent and the least-squares method, is introduced and the issue of how the equivalent FIS can be rapidly trained and adapted with this algorithm is discussed.
As a simple example, a FIS with two inputs x and y and one output z is assumed. The first order Sugeno fuzzy model, a typical rule set with two fuzzy if- then rules can be expressed as:

Rule 1: If x is A1 AND y is B1 THEN z1= p1x+q1y+r1
Rule 1: If x is A2 AND y is B2 THEN z2= p2x+q2y+r2

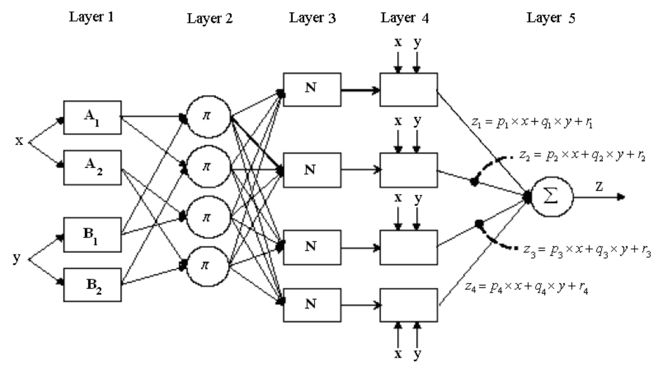

Fig.3 The schematic of ANFIS model structure

1st, 2nd and 5th layers are respectively similar to fuzzification, fuzzy operation and defuzzification in FIS. In 3rd layer average rules is computed by Eq. (14);

$$\overline{w}_i = \frac{w_i}{\sum_{i=1}^{n} w_i} \quad ,i = 1,2,... \qquad (14)$$

The 4th layer is named as Rules layer. The contribution of ith rule towards the total output or the model output and/or the function defined is calculated by Eq. (15);

$$z_i = \overline{w}_i f_i = \overline{w}_i (p_i x + q_i y + r_i) \quad , \quad i = 1,2 \qquad (15)$$

The objective is to train adaptive networks for having convenient unknown functions given by training data and finding the proper value of the input and output parameters. For this aim, ANFIS applies the hybrid-learning algorithm, consists of the combination of the “gradient descend” and “the least-square” methods. The gradient descent is employed to identify the linear output parameters (pi, qi, ri).
In this study for valuation of accuracy of models tree criterions (coefficient of determination, coefficient of efficiency and root-mean-square error) were used. Note that the coefficient of determination (R2) can be calculated as

$$R^2 = \frac{\sum_{i=1}^{N}(Q_{mi}-\overline{Q}_m)(Q_{pi}-\overline{Q}_p)}{\left(\sum_{i=1}^{N}(Q_{mi}-\overline{Q}_m)^2\right)^{0.5}\left(\sum_{i=1}^{N}(Q_{pi}-\overline{Q}_p)^2\right)^{0.5}} \qquad (16)$$

At coefficient of efficiency (E) account for model errors in estimating the mean or variance of the observed data sets. E Range from minus infinity (poor model) to 1.0 (perfect model), and can be calculated as

Table 1 Statistical characteristics of river flow data series

Statistic parameter	Q (m3/s)
Number of datapoints	7303
Mean	2.43
Maximum value	39.6
Minimum value	0
Standard deviation	3.366
Skew	3.686
Kurtosis	20.65

$$E = 1.0 - \frac{\sum_{i=1}^{N}(Q_{mi}-Q_{pi})^2}{\sum_{i=1}^{N}(Q_{mi}-\overline{Q}_m)2} \qquad (17)$$

In other to objectively evaluate the model performance, the most commonly employed error measure, such as the root-mean-square error (RMSE) was computed for models.

$$RMSE = \sqrt{\sum_{i}^{N}\frac{(Q_{mi}-Q_{pi})^2}{N}} \qquad (18)$$

Qm	Measured runoff [L3/T]
$\overline{Q}_m$	Measured mean runoff [L3/T]
Qp	Predict runoff [L3/T]
$\overline{Q}_p$	Measured mean runoff [L3/T]
N	Number of observation

4 Case study:

Aharchay drainage basin of the Aras River sub basins with the equivalent area of 2232 kilometers, a significant portion of Aras basin in Iran covers. The basin between the coordinates 30 ° 46 to 40 ° 47 east

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longitude and from 20 ° 38 north latitude to 45 ° 38 is located. The average height of the area 1880 meters above sea level and mean basin slope is 22 percent. Aharchay River basin as the main drain is mentioned. Figure (4) shows general map of Aharchay watershed. The daily discharge has been measured for almost 19 years, 7303 days. The daily runoff time series for 19 years, which were used in this research, are presented in Fig. 5. The statistical parameters of the flow data for Orang stations are given in Table 1.

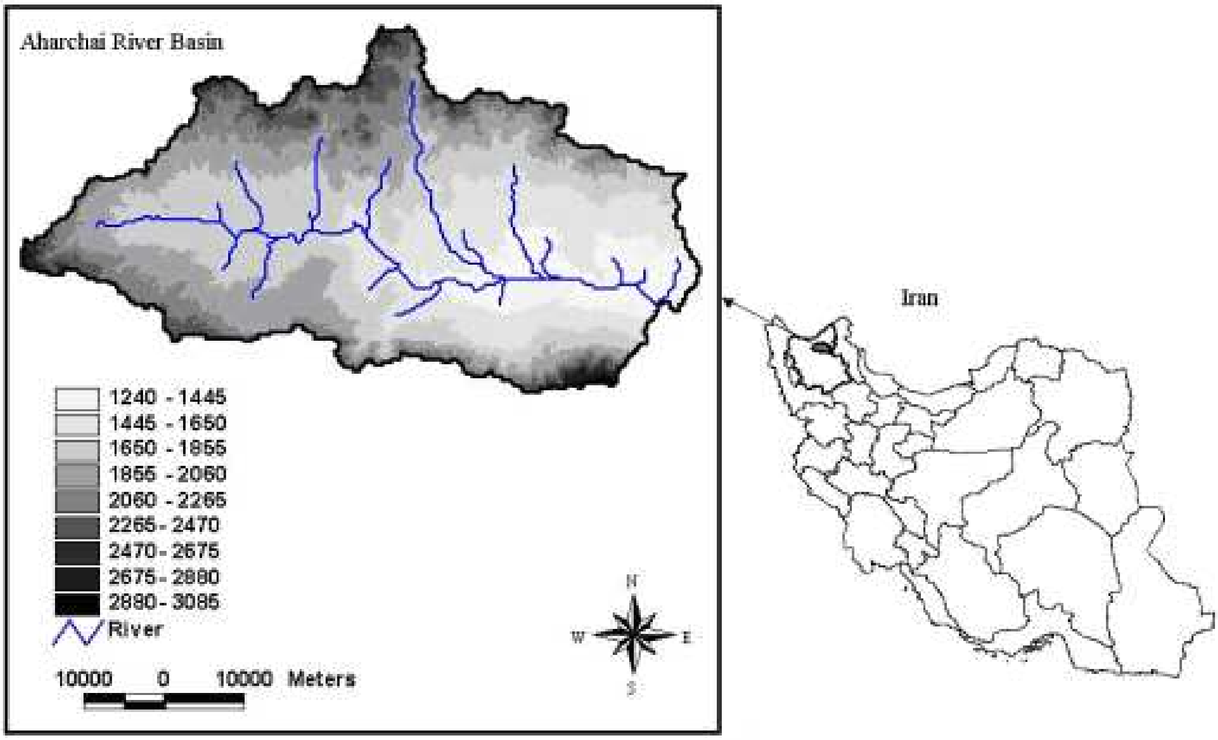

Fig. 4 Location of Study area

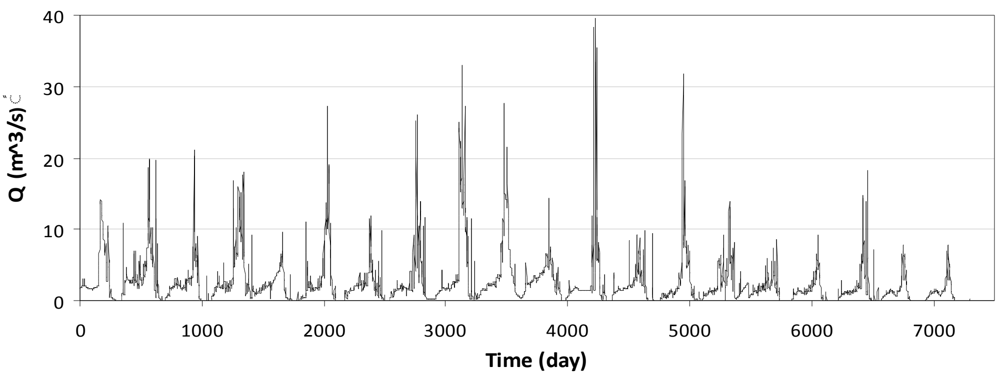

Fig. 5 Time series plot for river flow at Orang station in Aharchai river basin

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5 Results

Identification of Low-dimensional Chaos in the Time Series

The 19 years dataset of Orang station is divided into two parts: (i) the first 18 years (1984- 2003) of data are used in the phase-space identification and (ii) the subsequent 1 year dataset (2004) is used for prediction; A visual assessment for the existence of chaotic behavior in the river flow time series may be obtained by the reconstruction of phase space diagram and a selection of results for those at Orang station are shown in Figure 6. This is suggestive of a possible existence low-dimensional chaos in both of the dataset, in which the narrow dark band signifies strong determinism and the scattered band signifies the presence of noise in the data.

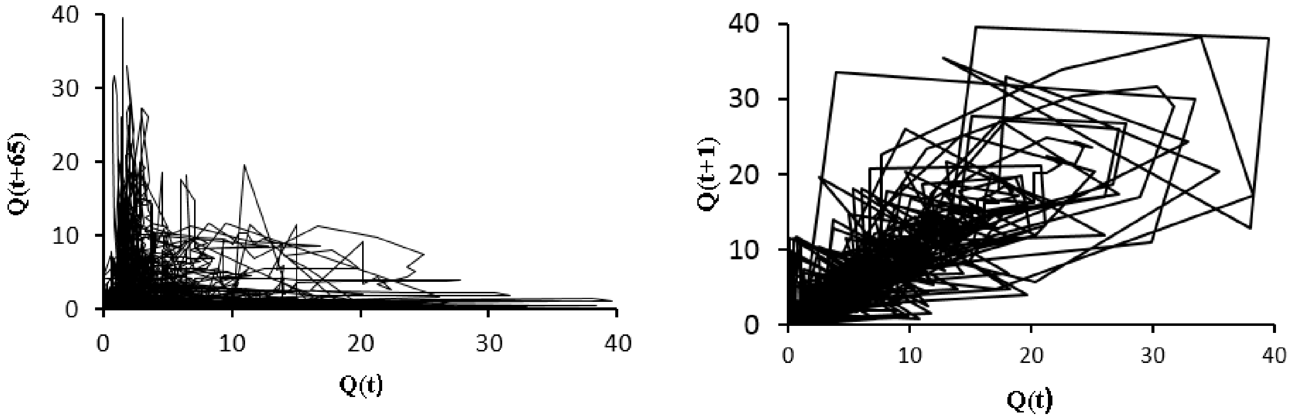

Fig. 6 Reconstruction of Phase Space for Time Series at Orang Station

Two methods are used to identify a possible existence of chaos in the river flow time series at Orang station. Using the AFC method, the delay time, τ is estimated for the time series at both of the stations at each timescale of the timeseries as the intercept with the x-axis of the curves by plotting the values of the ACF evaluated by the TISEAN package (Hegger et al., [32]) against delay times progressively increased from 1 to 100. The values of delay times are obtained as the zero intercepts of ACF and selected results are presented in Figure 7 for Orang station.

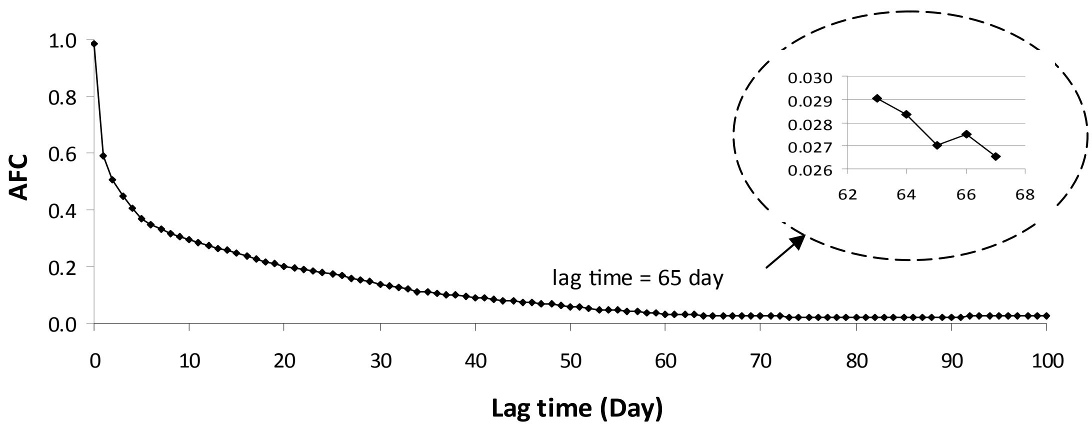

Fig.7 Autocorrelation Function – Sample Results for Orang Station

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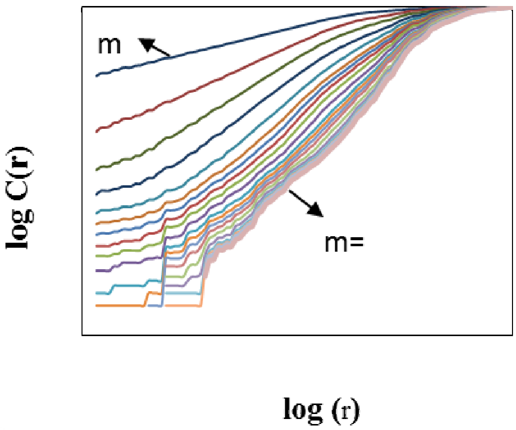
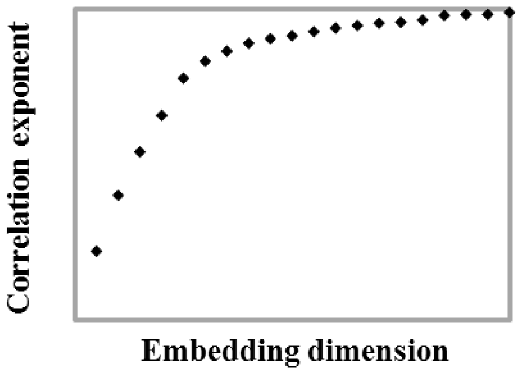
The correlation function method is implemented by setting the embedding dimension values, m, from 1 to 20 and varying systematically the values of, r, from a low value to, say, 100. The result is shown in Figure 8 for Orang station. The method identifies the existence of chaos in the following ways: (i) By plotting logC(r)/log(r) versus log(r), the function values tend to fluctuate at low values of radius r, signifying their stochastic strength, but for higher values of r the function tends to find a plateaux, where the values of logC(r)/log(r) becomes saturated for each timescale value providing visual evidence for a deterministic behaviour. (ii) The behaviour of correlation function *C*(r) against radius r for values of increasing m for the time series, providing further evidence for deterministic chaos if the correlation function $C(r)$ converges towards a single point underpinning the role of deterministic processes. (iii) The values D2(m) increase with increasing the embedding dimension values, m, up to a certain
value but the existence of chaotic behaviour is only underpinned if the values D2(m) saturate by reaching a plateaux.



(a)



(b)

Fig.8 Results of AFC: (a) Log C(r) Versus Log(r) for m= 1-20; (b) Log C(r) versus Log(r) for m=1-20

Results by Local Prediction and ANFIS Models

Local prediction algorithm is used to predict river flow time series at Orang station. The procedure involves varying the value of the embedding dimension in a range, say 2 – 14, and estimating the value of correlation coefficient (R) and Root Mean Square Error (RMSE). The embedding function with the highest coefficient of correlation is selected as the solution. This is given in Table 2 for the dataset with daily time interval, as well as a selection of other time steps. It show that the best prediction is achieved when the embedding dimension is Delay time=65 day and mopt=2. These values also comply with recommendations by: (i) Farmer and Sidorowich [33], Abarbanel et al. [34] suggesting an embedding dimension just greater than the attractor dimension ($m > D_2$; which is 5). (ii) Takens [18] suggesting an embedding dimension, m ($m \geq 2D_2 + 1$, which is 11).

Table 2 Local Prediction Using Different Embedding Dimension for River

m	2	3	4	5	6	7	8	9	10	11	12	13	14
R2	0.969	0.969	0.954	0.966	0.945	0.945	0.959	0.949	0.944	0.938	0.897	0.916	0.838
E	0.937	0.935	0.908	0.931	0.885	0.916	0.894	0.883	0.868	0.764	0.820	0.721	0.679
RMSE	0.122	0.125	0.177	0.134	0.222	0.162	0.204	0.225	0.255	0.457	0.348	0.517	0.621

In this paper are performed ANFIS models. Different this models is in number of inputs. In First model, ANFIS input is Q (t-1) and in second model, ANFIS inputs are Q(t-1) and Q(t-2) and in third model, ANFIS inputs are Q(t-1), Q(t-2) and Q(t-3).The results of ANFIS models are presented in table 3. Also Comparing Tables 2 and 3 shows the ANFIS model is more than of the Chaos model.

According to Table 3 it gives, accuracy of ANFIS model with single input variable is more than other, although difference between them results are low. ANFIS is a very powerful tool for dealing quickly and efficiently with imprecision and nonlinearity. Anfis and Chaos models time series is shown in Fig 9.

Table 3 Result of ANFIS model with different input variable

Input variable	Output	R2	E	RMSE
Q(t-1)	Q(t)	0.992	0.979	0.486
Q(t-1), Q(t-2)	Q(t)	0.981	0.959	0.678
Q(t-1), Q(t-2),Q(t-3)	Q(t)	0.904	0.912	1.444

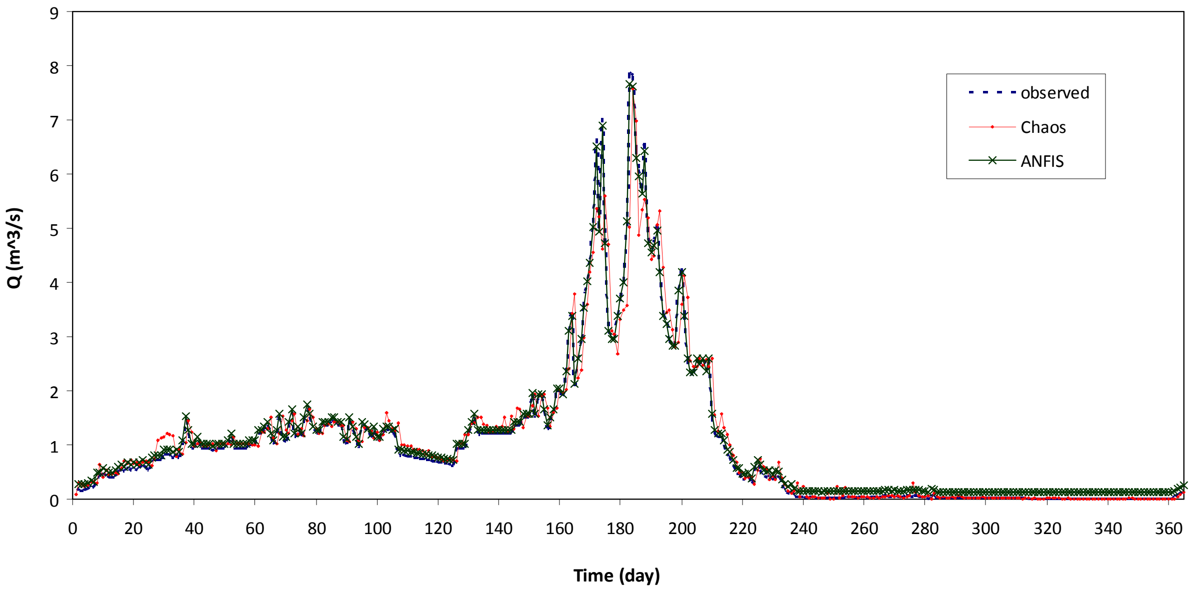


Fig.9 Comparison of ANFIS Prediction with the Local Prediction Model

The comparative performance of the models is shown the scatter plot of observed and calculated values in Fig 10. The scatter diagrams for both models show very close results.

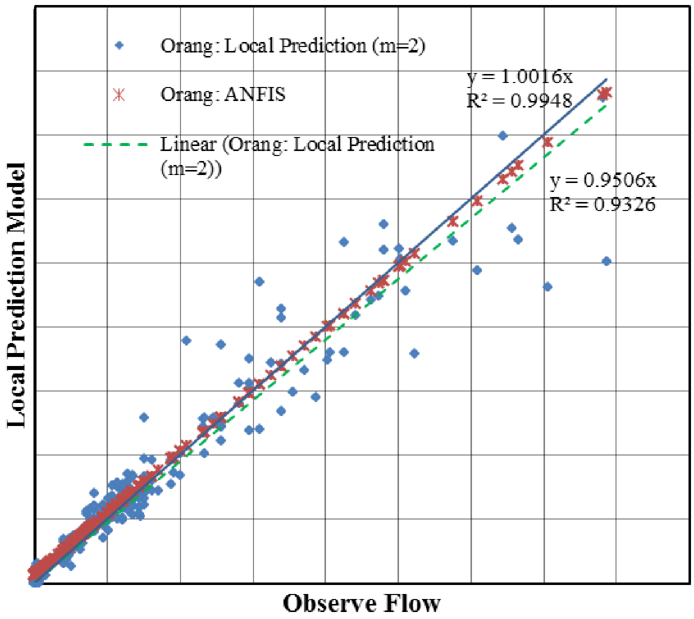


Fig.10 Scatter Diagram for the Models at Orang Station

6 Conclusions

Chaos theory with quantum theory and relativity is one of the most important discoveries of The last century.with review of time series caused of dynamical systems, such as behavior of rivers by chaos theory can predicted behavior of system.

Dimensionality of a time series represents the level of complexity of the underlying system dynamics (and number of dominant governing variables), and therefore the above mentioned nonlinear dynamic- and dimensionality-based classification certainly helps in identifying the appropriate structure and complexity of models. Sivakumar and Singh have classified streamflow in the western United States according to correlation dimension to 4 classes [2].

The nearest integer above the saturation value, fractals dimension, is generally considered to provide the minimum number of phase-space or variables necessary to model the dynamics of the attractor. The value of the embedding dimension at which the saturation of the correlation exponent occurs generally provides an upper bound on the number of variables sufficient to model the dynamics [35]. Results of this study (D2=3.5) indicates low chaotic and less complex system. Number of variables necessary for model is 4 (fractals dimension=4).

Both chaotic and Anfis techniques perform equally well for the river flow time series studies here but chaos theory give more information about river flow time series.

References

[1] Sivakumar B (2001) Is a chaotic multi-fractal approach for rainfall possible?. Hydrological Processes, 15: 943-955.

[2] Islam M. N, Sivakumar B (2002) Characterization and prediction of runoff dynamics: A nonlinear dynamical view. Advances in Water Resources, 25(2): 179-190.

[3] Sivakumar B, Singh V.P (2012) Hydrologic system complexity and nonlinear dynamic concepts for a catchment classification framework. Hydrology and Earth System Sciences, 16: 4119–4131.

[4] Domenico M. De, Ghorbani M. A (2011) Chaos and scaling in daily river flow. Chaos, Solitons & Fractals, 8 April.

[5] Nagesh Kumar D, Dhanya, C.T. (2011) Predictability and Chaotic Nature of Daily Streamflow. 34th IAHR World Congress - Balance and Uncertainty, Brisbane, Australia, 1427-1434.

[6] Ghorbani M. A, Kisi O, Aalinezhad. M (2010) A probe into the chaotic nature of daily streamflow time series by correlation dimension and largest Lyapunov methods. Applied Mathematical Modelling 34: 4050–

4057
[7] Sivakumar B, Jayawardena A. W, Fernando T. M. G. H (2002) River flow forecasting: Use of phase-space reconstruction and artificial neural networks approaches. J Hydrology 265(1-4): 225-245.
[8] Lisi F, Villi V (2001) Chaotic forecasting of discharge time series: A case study. J the American Water Resources Association 37(2): 271-279.
[9] Liu Q, Islam S, Rodriguez-lturbe I, Lee Y (1998) Phase-space analysis of daily streamflow: characterization and prediction. Advances in Water Resources 21: 463-475.
[10] Stehlik J (1999) Deterministic chaos in runoff series. J Hydrology and Hydrodynamics 47 (4): 271-287.
[11] Regonda S. K, Sivakumar B, Jain A (2004) Temporal scaling in river flow: can it be chaotic? Hydrological Sciences–Journal–des Sciences Hydrologiques 49 (3), 373-385.
[12] Ghorbani M. A, Khatibi R, Aalami M. T, Kocak K, Makarynskyy O, Makarynska, Aalinezhad M (2011) Dynamics of hourly sea level at Hillarys Boat Harbour, Western Australia: a chaos theory perspective. Ocean Dynamics, DOI 10.1007/s10236-011-0466-8.
[13] Siek M, Solomatine D. P, (2010) Nonlinear chaotic model for predicting storm surges. J Nonlinear Processes in Geophysics, 17: 405–420.
[14] Khokhlov V, Glushkov A, Loboda N, Serbov N, Zhurbenko K (2008) Signatures of low-dimensional chaos in hourly water level measurements at coastal site of Mariupol, Ukraine. J Stochastic Environ Res and Risk Assessment 22: 777–787.
[15] Skebede S, Travi Y, Alemayehu T. and Marc V., (2005) Water balance of lake Tana and its sensitivity to fluctuations in rainfall, Blue Nile Basin, Ethiopia, J Hyd: 1-15.
[16] Bayram B, Bayraktar H, Helvaci C, and Acar U. (2004) Coast line change detection using corona, SPORT and IRS ID Images, Turkey-Istanbul.
[17] Solomatine D. P, Velickov S, and Wust J. C (2001) Predicting Water Levels and Currents in the North Sea Using Chaos Theory and Neural Networks. Proc, 29th Iahr Congress, Beijing, China, 1-11.
[18] Takens F (1981) Detecting strange attractors in turbulence, Lectures Notes in Mathematics, In: Rand D. A, Young L. S. (Eds.), 898: 366-381. Springer-Verlag, New York.
[19] Holzfuss J, Mayer-Kress G (1986) An approach to error-estimation in the application of dimension algorithms, In: Mayer-Kress G, editor. Dimensions and entropies in chaotic systems, New York: Springer 114–22.
[20] Fraser A. M, Swinney H. L (1986) Independent coordinates for strange attractors from mutual information. Physical Review A, 33(2): 1134-1140.
[21] Liebert W, Schuster H.G (1989) Proper choice of time delay for the analysis of chaotic time series. Phys. Lett. A, 142 : 107-1111
[22] Schuster HG (1988) Deterministic chaos. Weinheim: VCH;
[23] Tsonis AA, Elsner JB. (1988) The weather attractor over very short timescales. Nature.333:545–547.
[24] Grassberger P, Procaccia I (1983) Measuring the Strangeness of Strange Attractors. Physica D: Nonlinear Phenomena 9 (1-2): 189-208.
[25] Itoh K (1995) A method for predicting chaotic time-series with outliers. Electron. Commun. Jpn., 78 (5) : 44–53
[26] Porporato A, Ridolfi L (1997) Nonlinear analysis of river flow time sequences. Water Resources Research 33(6): 1353-1367.
[27] Koçak K (1997) Application of local prediction model to water level data. A satellite Conference to the 51 st ISI Session in stanbul, Turkey. Water and Statistics, Ankara-Turkey, 185-193.
[28] Jang J. S, Sun C. T, Mizutani E (1997) Neuro-fuzzy and soft computing: A computational approach to learning and machine intelligence. Prentice - Hall International. New Jersey.
[29] Zadeh L. A, (1965) Fuzzy sets, information and Control. 12(2): 94-102.
[30] Tayfur G, Singh V. P (2006) ANN and Fuzzy Logic for simulating event-baised rainfall-runoff. J Hydrologic Eng, ASCE, 132(12): 1321-1329
[31] Ross T. J, (1995) Fuzzy logic with engineering application. McGraw Hill, Inc., USA.
[32] Hegger R, Kantz H, Schreiber T (1999) Practical implementation of nonlinear time series methods: The TISEAN package. J Chaos 9(2): 413-435.
[33] Farmer J. D, Sidorowich J. J, (1987) Predicting chaotic time series. Physical Review Letters 59 (8): 845–848.
[34] Abarbanel H. D. I, (1990) Prediction in chaotic nonlinear systems: methods for time series with broadband Fourier spectra. Physical Review A. 41 (4): 1782–1807.
[35] Sivakumar B, Berndtsson R, Olsson J and Jinno K (2001) Evidence of chaos in the rainfall-runoff process. J Hydrological Sci-J-des Sci Hydrologiques, 46(1) Febr.