A New Method for Detecting the Number of Coherent Sources in the Presence of Colored Noise

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Abstract

In this paper, a new method for determining the number of coherent/correlated signals in the presence of colored noise is proposed which is based on the Eigen Increment Threshold (EIT) method. First, we present a new approach which combines EIT criterion and eigenvalue correction. The simulation results show that the new method estimates the number of noncoherent signals in the presence of colored noise with higher detection probability respect to MDL, AIC, EGM and conventional EIT. In addition, to apply the proposed EIT algorithm to detect the number of sources in the case of coherent and/or correlated sources, a spatial smoothing preprocessing is added. In this case, simulation results show 100% detection probability for signal to noise ratios greater than -5dB. Final version of the proposed EIT-based method is a simple and efficient way to increase the detection probability of EIT method in the presence of colored noise considering either coherent/correlated or noncoherent sources.

Keywords: Eigen Increment Threshold, Colored Noise, Coherent Signal, Eigenvalue Correction, EIT, EGM.

1. Introduction

Array signal processing can be applied to many applications, such as radar, sonar, remote sensing, astronomy, seismology and mobile communications. In this field, estimating direction of arrival (DOA) of signals in an array of sensors, has received a considerable amount of attention during the last two decades [1]. Most of the existing estimation techniques, assume that the number of sources is known a priori and may give misleading results if the wrong number of sources is considered. Therefore, to achieve good results in DOA estimation, it is a key problem to determine the number of signals correctly [2, 3].

There are a number of different methods to detect the number of sources. Akaike's information criterion (AIC) and minimum description length (MDL) are the two most popular methods that detect the number of signals based on information theoretic criteria [4]. According to these algorithms, the number of signals is determined as the value for which the AIC or MDL criteria, is minimized [5]. Recently, these methods have been studied in [6] for source enumeration and most of these research devoted to modify this approach. For example, their methods cannot resolve coherent sources. In [7], a new approach has been proposed that can resolve coherent sources. The method uses the MDL principle, and decomposes data into signal and noise components. The MDL descriptor is then computed for signal and noise components separately, and the results are added to obtain the total MDL cost.

In addition to MDL and AIC methods, some other algorithms that consider a bound or threshold for eigenvalues are proposed. Chen et al. [8] proposed a method for determining the number of sources by setting an upper bound on the values of the eigenvalues. This bound is determined by an adjustable parameter, so its performance is better than MDL at low SNRs (signal to noise ratios), and better than AIC at high SNRs. Zhang et al. [9] offered a set of eigenvalue Gradient methods (EGMs), which like AIC and MDL methods, it also enumerate the number of non-coherent sources according to the eigenvalues of auto-correlation matrix.

In [10], the performance of three popular methods, AIC, MDL and EGM is evaluated. Simulation results for determining the number of noncoherent sources show that EGM offers higher performance with respect to MDL and AIC, in low SNRs. Moreover, simulation results for smoothed algorithms, in the case of coherent/ correlated sources, show higher detection probability for EGM compared to AIC and MDL.

Due to higher performance of Eigen Increment Threshold (EIT) method with respect

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to the AIC and MDL methods, it is widely adopted. In contrast, the energy will be shifted to the biggest eigenvaule and it directly leads to increase EIT threshold and the occurrence of under-estimation when unequal power sources exist. To overcome this phenomenon, the modified EIT suggested in [11] is appropriate which uses a recursive method to avoid the influence of the bigger eigenvalue. Considering this modification, the detection probability can be evidently increased in the case of unequal power sources.

All of the above mentioned algorithms, such as: MDL, AIC, EGM and EIT have been derived based on the assumption that the noise of the sensors is white. In practice, signals experience colored noise which causes a rapid decrease in the performance of the conventional methods.

Jing and Rong [12] addressed a novel effective accurate detection algorithm in spatial color noise by using the canonical correlation coefficients of the joint covariance matrix. Compared with the other algorithms, this algorithm has a better performance and lower complexity. Recently, Zhen et al. [13] established a new method to eliminate the inequality of the noise eigenvalues caused by colored noise for coherent signals. It combines the information theoretic criteria and eigenvalue correction to estimate the number of coherent signals.

In this paper, a novel algorithm to detect the number of sources in the presence of colored noise is proposed. The approach is based on a combination of EIT and eigenvalue correction. Since conventional EIT can detect only the number of noncoherent sources, we use forward/backward spatial smoothing (FBSS) technique [14,15] to determine the number of coherent signals. The simulation results demonstrate the effectiveness of the proposed algorithm and show that the presented method offers higher performance and lower complexity in a joint state. It means that the proposed algorithm can be used to detect the number of coherent/correlated sources in the presence of colored noise.

This paper is organized as follows. After the statement and formulation of the problem in Section 2, Section 3 describes the details of the EIT method for non-coherent sources in the presence of white noise. Section 4 presents a new algorithm as a combination of eigenvalue correction technique and EIT for determining the number of sources in colored noise. Section 5 presents FBSS technique as a preprocessing step for determining the number of coherent/correlated sources. Section 6 demonstrates the simulation results. Finally, Section 7 concludes this research.

2. Signal Model

Suppose that P narrowband signals, emitted from the far field impinging on array of M sensors $(M > P)$. The observed data from M sensor array, can be described as follows

$$x(t) = As(t) + n(t) \qquad t = 1,2, \dots, N \qquad (1)$$

where $A = [a(\theta_1)a(\theta_2) \dots a(\theta_P)]$ is the matrix of array manifold, $s(t) = [s_1(t)\, s_2(t) \dots s_P(t)]^T$ is the vector of signal, and $n(t) = [n_1(t)n_2(t) \dots n_M(t)]^T$ is the vector of noise. Noise sequence $n_1(t), n_2(t), \dots, n_M(t)$ is colored Gaussian noise which is zero mean and uncorrelated with the signals [12]. The covariance matrix for array output is

$$R = E[x(t)x^H(t)] = AR_sA^H + R_n \qquad (2)$$

where $R_s = E\{s(t)s^H(t)\}$ and $R_n = E\{n(t)n^H(t)\}$ are the correlation matrix of signal and noise, respectively. In practice, considering ergodic processes, correlation matrix is acquired from limited data, that is

$$\widehat{R} = \frac{1}{N}\sum_{t=1}^{N} x(t)x^H(t) \qquad (3)$$

where N is the number of snapshots.

To simplify the presentation and also determining the number of sources from $\widehat{R}$, we consider the following assumptions [11].

- The array manifold, defined as the set of steering vectors is known.
- Any subset of M steering vectors from the array manifold is linearly independent.
- Arriving signals $s_i(t)$ are stationary narrowband signals with similar center frequencies.
- The number of sensors is greater than the number of sources, namely $M > P$.

3. Detecting the Number of Sources using EIT

Just like EGM method, EIT also determines the number of noncoherent sources according to the eigenvalues of auto-correlation matrix. Therefore, after calculating the spatial auto-correlation matrix of the output data $x(t)$ of the sensor array by Eq. 3, we arrange the eigenvalues in descending order.

$$\lambda_1 \geq \cdots \geq \lambda_P \geq \lambda_{P+1} \geq \cdots \geq \lambda_M \qquad (4)$$

There is a significant difference between λ_P and λ_{P+1}. Hence, by checking the difference between neighbor eigenvalues and setting an upper bound on the values of the eigenvalues, the number of P signals can be detected. This threshold is given as [11]:

$$\lambda_i - \lambda_{i+1} \leq \text{threshold} \equiv \eta(M,N) \frac{P_s}{\left(1+\sqrt{P_s/\lambda_M}\right)^2} \quad (5)$$

$i = M-1, M-2, \ldots, 1$

where P_s is the power of source, and $\eta(M,N)$ is a coefficient function with following features:

- $\eta(M,N) = 1$ when $N \to \infty$
- $\eta(M,N)$ increases when N reduces
- $\eta(M,N)$ decreases when M increases

By finding out all i that satisfy $\Delta\lambda_i \leq$ threshold to construct the set $\{i_k\} = \{i|\Delta\lambda_i \leq \text{threshold}\}$ and taking $i_0 = \min\{i_k\}$, the estimated number of signals is $\hat{p} = i_0 - 1$.

4. New Eigenvalue corrected EIT-based Technique

EIT has a good performance in the presence of white noise, but capability of this algorithm will be sharply declined when signals experience colored noise. Therefore, for smoothing the spatial colored noise, we can use eigenvalue correction method which generates best correction values to getting properly results. "Colored noise cannot be effectively smoothed if the correction eigenvalue is too small. In other hand, not only noise eigenvalue but also signal eigenvalue can be affected if the correction eigenvalue is too large". The eigenvalues of $\hat{R}$ given by Eq. 4, and e_i is correction value of $\lambda_i (i = 1,2,\ldots,M)$. In order to smooth colored noise effectively, e_i is defined as [13]:

$$e_i = \sqrt{\sum_{j=1}^{i} \lambda_j} \qquad i = 1,2,\ldots M \quad (6)$$

Consequently, the corrected eigenvalue named $c_i (i = 1,2,\ldots,M)$, can be obtained by:

$$c_i = \lambda_i + e_i \quad (7)$$

For each eigenvalue, related new corrected eigenvalue will be calculated. Suppose that there are k signals, we can get M eigenvalues after decomposing covariance matrix, that is

$$\lambda_1 \geq \cdots \geq \lambda_P \geq \lambda_{P+1} \geq \cdots \geq \lambda_M \quad (8)$$

Using Eq. 6, they become corrected as $\tilde{\lambda}_1 \geq \cdots \geq \tilde{\lambda}_P \geq \tilde{\lambda}_{P+1} \geq \cdots \geq \tilde{\lambda}_M$. where

$$\tilde{\lambda}_i = \lambda_i + \sqrt{\sum_{j=1}^{i} \lambda_j} \qquad i = 1,2,\ldots,M \quad (9)$$

Substituting Eq. 9 into Eq. 5, threshold function becomes:

$$\tilde{\lambda}_i - \tilde{\lambda}_{i+1} \leq \text{threshold} \equiv \eta(M,N) \frac{P_S}{\left(1+\sqrt{P_S/\tilde{\lambda}_M}\right)^2} \quad (10)$$

$i = M-1, M-2, \ldots, 1$

By finding out all i that satisfy $\Delta\tilde{\lambda}_i \leq$ threshold and taking out $i_0 = \min(i_k)$, someone can get the number of signals as $\hat{p} = i_0 - 1$.

5. Forward/ Backward Spatial Smoothing Technique

Spatial smoothing is based on a preprocessing scheme that divides the total array of M sensors, into overlapped sub-arrays of size M_0 (see Fig. 1) and then, generates the average of the sub-array output covariance matrices. Let $x_l^f(t)$ stands for the output of the lth sub-array for $l = 1,2,\ldots L \equiv M - M_0 + 1$ where L denotes the total number of these forward sub-arrays [14].

$$x_l^f(t) = [x_l(t), x_{l+1}(t), \ldots, x_{l+M_0-1}(t)]^T \quad (11)$$

Then, the covariance matrix of the lth sub-array is

$$R_l^f = E\left[x_l^f(t)(x_l^f(t))^H\right] \quad (12)$$

Forward spatially smoothed covariance matrix R^f as the mean of the forward sub-array covariance matrices is

$$R^f = \frac{1}{L}\sum_{l=1}^{L} R_l^f \quad (13)$$

The covariance matrix of the lth backward sub-array is given by

$$R_l^b = E\left[x_l^b(t)(x_l^b(t))^H\right] \quad (14)$$

In the same way as Eq. 13, the spatially smoothed backward sub-array covariance matrix R^b is

$$R^b = \frac{1}{L}\sum_{l=1}^{L} R_l^b \quad (15)$$

Finally, the forward/backward smoothed covariance matrix $\tilde{R}$ as the mean of R^f and R^b is given by

$$\tilde{\mathbf{R}} = \frac{\mathbf{R}^f + \mathbf{R}^b}{2} \quad (16)$$

The smoothed covariance matrix $\tilde{R}$ in Eq. 16 has exactly the same form as the covariance matrix for some non-coherent situations. It means that $\tilde{R}$ can be used instead of $\hat{R}$. The eigenstructure-based techniques can be applied to this smoothed covariance matrix, to successfully estimate the number of coherent sources [14].

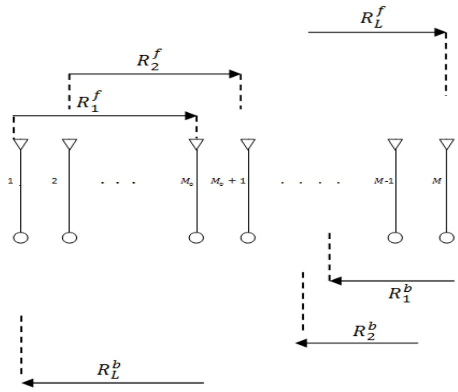

Fig 1. The forward/backward sub-arrays in FBSS scheme [14].

Finally, the Proposed EIT-based algorithm which determines the number of coherent sources in the presence of colored noise is summarized as follows:

Table 1: Summary of the proposed FBSS-based EIT algorithm

Step1.	Compute the smoothed covariance matrix $\tilde{R}$ by: $\tilde{R} = \frac{R^f + R^b}{2}$ Then, get M eigenvalues after decomposing it.
Step2.	Use Eq. 6 to correct M eigenvalues. Corrected eigenvalues become $\tilde{\lambda}_1 \geq \cdots \geq \tilde{\lambda}_P \geq \tilde{\lambda}_{P+1} \geq \cdots \geq \tilde{\lambda}_M$
Step3.	Substitute corrected eigenvalues into Eq. 5. The revised threshold function can be derived and threshold inequality as 10 should be checked:
Step4.	$\hat{p} = i_0 - 1$

6. Conclusions

In this research, the effectiveness of conventional and the proposed EIT algorithm is compared with MDL, AIC and EGM in different cases, numerically. All simulations are run in MATLAB software and associated figures are extracted from simulation results of this investigation. In Figure 2, the performance of the proposed EIT equipped with eigenvalue correction method is compared with EGM and conventional EIT methods. Both methods of Figure 3 are the new proposed ones. Finally, the simulation results of Figure 4 are reported for the first time in this research.

Consider a linear array of 8 sensors, exposed to the two non-coherent signals arriving from 0° and 10°. The inter-element spacing is half of the wavelength.

In the first experiment, the performance of the proposed EIT-based algorithm is compared with conventional EIT and EGM in the case of colored noise. Spatial colored noise is obtained by filtering white noise through a first order auto-recursive (AR1) filter, given by

$$y(i) = \alpha y(i-1) + x(i) \qquad (17)$$

where $\alpha \in [0,1]$ is the noise correlation coefficient of adjacent sensors.

In this experiment, $\alpha = 0.6$, $\eta(M,N) = 2$ and the number of snapshots is 200. The simulation results as depicted Fig. 2 show that despite the conventional EIT fails to detect the correct number of sources in the presence of colored noise, the proposed eigenvalue corrected EIT-based algorithm offers 100% success for SNRs greater than $-1dB$. In addition, for SNRs lower than $2dB$, the proposed method has a much better detection performance than the EGM algorithm.

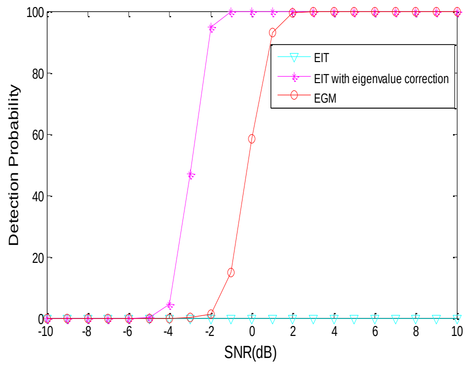

Fig 2. The detection probability versus SNR for experiment 1.

In the second experiment, noncherent signals with equal power impinge on array at 0° and 10°, SNR=10dB, the number of snapshots is 200 and noise correlation coefficient changes from 0 to 1 with step size 0.1. As shown in Fig. 3, for all correlation coefficients of noise, the conventional EIT method is completely ineffective. In contrast, the proposed EIT-based algorithm can offer 100% detection probability for $\alpha \in [0.1, 0.8]$.

In this part, spatial smoothing method is applied to solve the coherency and the number of coherent sources is estimated considering the smoothed output signal of array antenna. As the experiment 3, the performance of the proposed FBSS-based EIT and EGM algorithms under different SNRs, for coherent sources and colored noise is investigated. Three coherent signals with equal powers impinge on array at 0°, 5° and 10°. Path coefficients for coherent signals are (0.7+0.7j), (0.6+0.5j) and (0.2+0.4j). ULA has 9 elements, $\alpha = 0.6$ and SNR changes from -10dB to 10dB with step size 2dB. Also, $M_0 = 7$ and the number of snapshots is 200.

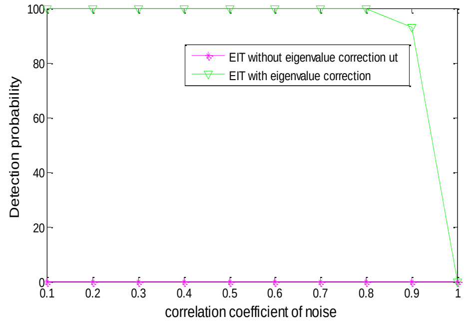

Fig 3. The detection probability versus noise correlation coefficient for experiment 2.

As illustrated in Fig. 4, the proposed FBSS-based EIT algorithm can determine the number of sources exactly when the SNR is greater than -3dB. In other word, using FBSS-based proposed EIT algorithm, SNR will be improved 9dB with respect to FBSS-based EGM.

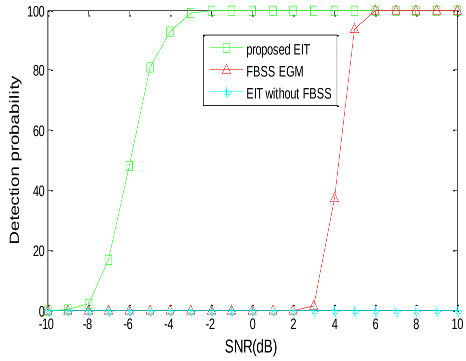

Fig 4. The detection probability versus SNR for experiment 3.

In order to show that the proposed method is computationally efficient, Table 2 lists the computational costs of two algorithms in terms of the number of multiplications/divisions, additions/subtractions, comparisons with a threshold, minimizations and logarithmic operators. It shows that the proposed method needs lower computational complexity with respect to the Zhen et al. [13] method. Here, M is the number of sensors and K is the number of signals.

Table 2: Comparison of computational complexity

Algorithm	Proposed EIT	Zhen et al. [13]
The number of Multiplications/ divisions	4	$M - K + 8$
The number of Additions/ subtractions	$3M + 1$	$3M - K + 4$
The number of Comparisons with a threshold	$M - 1$	None
Minimizations	None	M
The number of Logarithmic operators	None	1

7. Conclusions

In this paper, a new EIT-based algorithm that combines the EIT criterion and eigenvalue correction is proposed which is able to determine the number of both noncoherent and coherent signals in colored noise. When the signal sources are coherent, conventional threshold-based methods, EGM and EIT, cannot separate signal and noise eigenvalues and are not able to detect the number of signal sources. In this research, to overcome the effect of coherency, forward-backward spatial smoothing method is used. Hence, in the first step, the smoothed covariance matrix is fed to eigenstructure-based techniques to successfully estimate the number of coherent sources. In the second step, the noise eigenvalues of the smoothed covariance matrix are corrected using the eigenvalue correction technique. Finally, in the case of colored noise, corrected eigenvalues are used to find the threshold.

The effectiveness of the proposed EIT-based algorithm was verified through numerical examples. Simulation results showed that the proposed method estimates well the number of coherent or noncoherent signals in the presence of colored noise. Moreover, it was shown that the proposed method needs lower computational complexity compared to the previous method.

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